Differential Beamforming for Uniform Circular Array with Directional Microphones

Weilong Huang, Jinwei Feng

Speech Lab, Alibaba DAMO Academy

yuankai.hwl@alibaba-inc.com, jinwei.feng@alibaba-inc.com

Abstract

Use of omni-directional microphones is commonly assumed in the differential beamforming with uniform circular arrays. The conventional differential beamforming with omni-directional elements tends to suffer in low white-noise-gain (WNG) at the low frequencies and decrease of directivity factor (DF) at high frequencies. WNG measures the robustness of beamformer and DF evaluates the array performance in the presence of reverberation. The major contributions of this paper are as follows: First, we extends the existing work by presenting a new approach with the use of the directional microphone elements, and show clearly the connection between the conventional beamforming and the proposed beamforming. Second, a comparative study is made to show that the proposed approach brings about the noticeable improvement in WNG at the low frequencies and some improvement in DF at the high frequencies by exploiting an additional degree of freedom in the differential beamforming design. In addition, the beampattern appears more frequency-invariant than that of the conventional method. Third, we study how the proposed beamformer performs as the number of microphone elements and the radius of the array vary.

Index Terms: microphone array, directional microphone, differential beamforming

1. Introduction

Microphone array for speech enhancement is an indispensable part in many hands-free communication systems or far-field speech recognition systems in noisy and reverberant environments. Beamforming with a microphones array is an effective technique to enhance the target signal from desired direction and suppress the interferences from undesired direction [1]. The beamforming algorithms can generally be categorized into two groups: adaptive beamformer and fixed beamformer. As an adaptive beamformer, generalized sidelobe canceller (GSC) with a fixed blocking matrix [2] or adaptive blocking matrix [3] is an efficient approach to implement the minimum variance distortionless response(MVDR) beamformer, thus suitable for use in real-time systems. As compared to adaptive beamfomers, fixed beamformers generally are more robust since they are not involved with the adaptation process. Furthermore, the fixed beamformer is an essential part in the GSC structure.

By virtue of the pioneering work from Benesty and Chen, etc [4, 5, 6, 7, 8, 9], differential microphone array (DMA), among all fixed beamformers, attracts a significant amount of attention recently in both academia and industry because it possesses a few advantages. Firstly, it can construct a relatively frequency-invariant beampattern, thus appropriate for speech signal processing. Secondly, it has the potential to obtain a large directivity factor (DF) with small and compact aperture [6]. As one type of DMA, circular differential microphone array (CDMA) features the ability to steer the beam eletronically to any direction with a similar directional gain and has been extensively studied [10, 11]. It is this research scope that this work falls in.

It is to the authors' knowledge that all of the CDMA designs published so far have assumed the use of omni-directional microphones. Although the robust CDMA design can improve the white noise gain (WNG) with the minimum-norm solution [4, 5, 10] by using more microphone elements than the order of CDMA, the WNG may still be relatively low, especially at the low frequencies, causing the well-known white noise amplification problem in the practical implementations. In addition, DF of the conventional CDMA usually degrades as the frequency increases. Lastly, beampattern also tends to deform at the high frequencies.

In the previous work [12], the measured directional pattern of microphone is exploited to optimize the MVDR-based solution for a 4-element cardioid microphone array employed in Microsoft Kinect for Windows. Based on the delay and sum algorithm, the directional elements for microphone array have been shown to be advantageous over the omni-directional elements [13]. However, theory is still lacking on how to exploit the directional microphones on the design of differential beamforming for the uniform circular array. In this paper, we will focus on the theoretical representation of directional microphone and propose an CDMA-based solution by utilizing the directional microphones, i.e., the circular differential directional microphone array (CDDMA). Through a comparative study between the conventional beamformer and the proposed beamformer, we show the new design can alleviate WNG and DF problems mentioned above in the conventional CDMA. The basic idea is that the directional microphone element itself already provides some degree of directivity, thus the microphone array based upon this type of element can be reasonably expected to outperform the conventional CDMA.

The rest of paper is organized as follows. First, the directional microphone is introduced; the signal model and performance measures commonly used for CDMA evaluation are briefly described. We will then elaborate the design of CDDMA and compare the performance to the conventional CDMA through various simulations in terms of WNG, DF and beampattern at different frequencies. Lastly, some conclusions will be drawn.

2. Signal Model

When classified by the directionality of microphone, the omni-directional element and directional element are widely used in the industry. Omni-directional microphone picks up sound with equal gain from all directions while the directional microphone does it predominantly from some specific directions. A comprehensive study on different microphones is made in the Eargle's book [14]. Considering some certain reasons, e.g., microphone

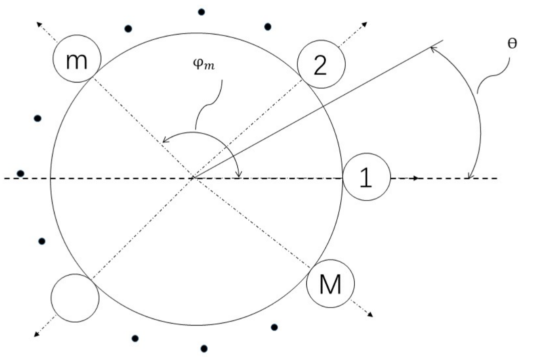


Figure 1: *Uniform circular array with directional microphones*

size or prices, the first-order directional microphone are often commercially used in a microphone array. Mathematically, the directional pattern of a first-order directional microphone can be expressed as $p+(1-p)\cos\alpha$, where α is the off-axis angle and p defines the property of the directional microphone [14]; for instance, it makes the well-known cardioid pattern when $p = 0.5$ and a dipole when $p = 0$. In this paper, we untilize the first-order directional microphone to illustrate the derivation and design examples.

There are two approaches to implement a first-order directional microphone: a dedicated directional microphone using a single microphone cartridge with two sound inlets and a two-omnidirectional-element system with some appropriate digital signal processing. It is known that the dedicated approach usually yields a better directional microphone in terms of signal-to-noise ratio (SNR) than the second approach [15]. This is because the signal processing that creates the directivity is done acoustically with the front and rear sound inlets. It is the dedicated directional microphone that we are going to utilize in our design. It should be also noted that the dedicated directional microphone comes in the form of either ECM (Electret Condenser Microphones) or MEMS (Micro-Electro-Mechanical System).

Now let us consider a single desired sound source in the far-field such that it appears as a plane wave impinging on a uniform circular array consisting of M first-order directional microphones with a radius of r. As depicted in Fig.1, the directional microphones are evenly distributed on the circle and pointed outward. The azimuth angle θ denotes the direction of arrival of the sound; c is the sound speed. In this scenario, the steering vector is given by

$$\mathbf{d}\left(\omega,\theta\right) = \left[d_1, d_2, \cdots, d_m, \cdots, d_M\right]^T \tag{1}$$

where the superscript ^{T} is the transpose operator and each element in (1) can be obtained as:

$$d_m = e^{j\frac{\omega r}{c}\cos(\theta-\varphi_m)}\left[p + (1-p)\cos(\theta-\varphi_m)\right] \tag{2}$$

where $j = \sqrt{-1}$ is the imaginary unit, $\omega = 2\pi f$ is the angular frequency, f is the temporal frequency, and $\varphi_m = \frac{2\pi(m-1)}{M}$ is the angular position of the m^{th} element. For comparison, we recall that the steering vector for conventional uniform circular array with omni-directional microphones is:

$$\mathbf{a}\left(\omega,\theta\right) = \left[a_1, a_2, \cdots, a_m, \cdots, a_M\right]^T \tag{3}$$

where $a_m = e^{j\frac{\omega r}{c}\cos(\theta-\varphi_m)}$. By examining the equations (1), (2) and (3), the steering vector $\mathbf{d}\left(\omega,\theta\right)$ is reformulated as:

$$\mathbf{d}\left(\omega,\theta\right) = \mathbf{u}\left(p,\theta\right) \circ \mathbf{a}\left(\omega,\theta\right) \tag{4}$$

where the operator $\circ$ is the Hadamard product, and we call $\mathbf{u}\left(p,\theta\right)$ as the directional microphone response vector for the direction of θ defined as below:

$$\mathbf{u}\left(p,\theta\right) = \left[u_1, \cdots, u_m, \cdots, u_M\right] \tag{5}$$

where $u_m = [p + (1-p)\cos(\theta-\varphi_m)]$ is the magnitude response of first-order directional microphone.

The problem of beamforming can be interpreted as a spatial filter to estimate the signal from desired "look" direction and suppress the signal from undesired direction by applying a complex weight vector:

$$\mathbf{h}(\omega) = \left[H_1(\omega)\ H_2(\omega)\ \cdots\ H_M(\omega)\right]^T \tag{6}$$

Given the signal model, the beamformer exhibits a distortionless response in the desired "look" direction $\theta_{desired}$, while in the undesired direction the beamformer shows a certain distortion in the response, i.e.,

$$\mathbf{d}^H(\omega,\theta)\mathbf{h}(\omega) \begin{cases} = 1, & \text{if } \theta = \theta_{desired} \\ < 1, & \text{if } \theta \neq \theta_{desired} \end{cases} \tag{7}$$

where the superscript ^{H} is the conjugate-transpose operator.

3. Performance Measures

For the sake of completeness, we now briefly introduce the mathematical definition of three widely-used performance measures for fixed beamforming, i.e., WNG, beampattern and DF. WNG shows the ability of a beamformer to suppress spatially uncorrelated noise [16]. It is also the most convenient way to evaluate the sensitivity of a beamformer to some of its imperfections such as sensor noise, position errors, etc [17]. Hence, WNG is also a reliable robustness measure [18]. WNG is defined as:

$$\mathcal{W}[\mathbf{h}(\omega)] = \frac{1}{\mathbf{h}^H(\omega)\mathbf{h}(\omega)} \tag{8}$$

Beampattern illustrates the directional sensitivity of a beamformer to a plane wave impinging on the array from the incident angle θ (see Fig.1):

$$\mathcal{B}[\mathbf{h}(\omega),\theta] = \mathbf{d}^H\left(\omega,\theta\right)\mathbf{h}(\omega) \tag{9}$$

In this paper, we utilize the power pattern, i.e., $|\mathcal{B}[\mathbf{h}(\omega),\theta]|^2$, to demonstrate the performance [19]. It is noted that the frequency-invariant beampattern is usually preferred for broadband signal processing. DF is defined as the ratio between the signal power in the array output in the desired steering direction and the power averaged over all directions[10, 19]:

$$\mathcal{DF}[\mathbf{h}(\omega)] = \frac{1}{\int_0^\pi d\phi \int_0^{2\pi} d\theta sin\phi \left|\mathcal{B}[\mathbf{h}(\omega),\theta,\phi]\right|^2} \tag{10}$$

where θ is the azimuth angle and the ϕ is the elevation angle; $\mathcal{B}[\mathbf{h}(\omega),\theta,\phi]$ is the beampattern in the spherical coordinate system, defined as:

$$\mathcal{B}(\mathbf{h}(\omega),\theta,\phi) = \mathbf{d}^H\left(\omega,\theta,\phi\right)\mathbf{h}(\omega) \tag{11}$$

where the mth element of the vector $\mathbf{d}(\omega, \theta, \phi)$ is:

$$[\mathbf{d}(\omega, \theta, \phi)]_m = e^{j\frac{\omega r}{c}\cos(\theta-\varphi_m)sin\phi} \\ [p + (1-p)\cos(\theta - \varphi_m)sin\phi] \quad (12)$$

Later in the analysis section, all the DF is calculated by numerical analysis based on the integral in the (10). We use this calculation instead of the analytical formula for omnidirectional microphone, since the theory of spherically isotropic noise field on directional microphones are lacking. And we use the directivey index to represent the DF as:

$$\mathcal{DI}[\mathbf{h}(\omega)] = 10log10(\mathcal{DF}[\mathbf{h}(\omega)]) \quad (13)$$

4. Proposed Beamformer

To design the proposed CDDMA beamformer, we formulate the problem as a linear system of equations as below

$$\mathbf{R}(\omega, \boldsymbol{\theta})\mathbf{h}(\omega) = \mathbf{c}_{\boldsymbol{\theta}} \quad (14)$$

where $\mathbf{h}(\omega)$ is the CDDMA beamforming weights we want to obtain and the constraint matrix $\mathbf{R}(\omega, \boldsymbol{\theta})$ of size $N \times M$ is given by

$$\mathbf{R}(\omega, \boldsymbol{\theta}) = \begin{bmatrix} \mathbf{d}^H(\omega, \theta_1) \\ \vdots \\ \mathbf{d}^H(\omega, \theta_N) \end{bmatrix}, \quad (15)$$

where $\mathbf{d}^H(\omega, \theta_n), n = 1, 2, \cdots, N$, is the steering vector of length M defined in (1), and

$$\boldsymbol{\theta} = [\theta_1 \; \theta_2 \; \cdots \; \theta_N]^T, \quad (16)$$

$$\mathbf{c}_{\boldsymbol{\theta}} = [c_{\theta_1} \; c_{\theta_2} \; \cdots \; c_{\theta_N}]^T, \quad (17)$$

are the vectors of size N containing the design parameters of the beamformer. $\theta_1, \cdots, \theta_N$ usually define the desired or null directions, and $c_{\theta_1}, \cdots, c_{\theta_N}$ are the corresponding response for these directions, e.g., given $c_{\theta_1} = 1$, the steering vector at θ_1 follows $\mathbf{d}^H(\omega, \theta_1)\mathbf{h}(\omega) = 1$, which means that θ_1 becomes the "look" direction of the proposed beamformer that produces the distortionless output for the sound coming from this direction. The remaining $c_{\theta_i} = 0$ for $i = 2, 3, ..., N$ are usually set to zero, i.e., these θ_i decides the nulls of the beampattern. Based on (14), we can see that the properties of the beamformer are determined by the constraint vector $\mathbf{c}_{\boldsymbol{\theta}}$ and the angle parameter vector $\boldsymbol{\theta}$ that need to be specified in the design. We will show different design examples by setting different constraint vectors $\mathbf{c}_{\boldsymbol{\theta}}$ in the latter part of the paper.

We take the well-known minimum-norm solution in [8, 10, 11] to solve our linear system equations as shown in (14), thus the CDDMA beamformer is obtained by:

$$\mathbf{h}_{cddma}(\omega) = \mathbf{R}^H(\omega, \boldsymbol{\theta})[\mathbf{R}(\omega, \boldsymbol{\theta})\mathbf{R}^H(\omega, \boldsymbol{\theta})]^{-1}\mathbf{c}_{\boldsymbol{\theta}}. \quad (18)$$

For comparison, we recall the CDMA beamformer with the minimum-norm solution based on the omnidirectional microphone signal model in [11] as:

$$\mathbf{h}_{cdma}(\omega) = \mathbf{A}^H(\omega, \boldsymbol{\theta})[\mathbf{A}(\omega, \boldsymbol{\theta})\mathbf{A}^H(\omega, \boldsymbol{\theta})]^{-1}\mathbf{c}_{\boldsymbol{\theta}} \quad (19)$$

where $\mathbf{A}(\omega, \boldsymbol{\theta})$ is the conventional far-field steering vectors for omnidirectional microphones defined as below:

$$\mathbf{A}(\omega, \boldsymbol{\theta}) = \begin{bmatrix} \mathbf{a}^H(\omega, \theta_1) \\ \vdots \\ \mathbf{a}^H(\omega, \theta_N) \end{bmatrix}, \quad (20)$$

The difference between the proposed beamformer and conventional CDMA beamformer is reflected in $\mathbf{R}(\omega, \theta)$ and $\mathbf{A}(\omega, \boldsymbol{\theta})$. Combining (4), (15) and (20), we obtain:

$$\mathbf{R}(\omega, \boldsymbol{\theta}) = \mathbf{U}(p, \boldsymbol{\theta}) \circ \mathbf{A}(\omega, \boldsymbol{\theta}), \quad (21)$$

where $\mathbf{U}(p, \boldsymbol{\theta})$ is called the directional microphone response matrix and expressed as:

$$\mathbf{U}(p, \boldsymbol{\theta}) = \begin{bmatrix} \mathbf{u}^H(\omega, \theta_1) \\ \vdots \\ \mathbf{u}^H(\omega, \theta_N) \end{bmatrix}. \quad (22)$$

Furthermore, combining (18) and (21), the CDDMA beamformer can be reformulated as:

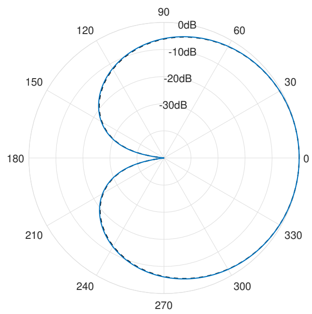
$$\mathbf{h}_{cddma}(\omega) = \mathbf{A}^H(\omega, \boldsymbol{\theta}) \circ \mathbf{U}^H(p, \boldsymbol{\theta}) \\ [(\mathbf{U}(p, \boldsymbol{\theta}) \circ \mathbf{A}(\omega, \boldsymbol{\theta}))(\mathbf{A}^H(\omega, \boldsymbol{\theta}) \circ \mathbf{U}^H(p, \boldsymbol{\theta}))]^{-1}\mathbf{c}_{\boldsymbol{\theta}}. \quad (23)$$

This equation shows neatly the relationship between the solutions of conventional CDMA and proposed CDDMA, i.e., CDDMA extends CDMA by introducing another degree of freedom, i.e., $\mathbf{U}(p, \boldsymbol{\theta})$. Put in a different way, CDMA is a special case of CDDMA when the microphone response matrix $\mathbf{U}(p, \boldsymbol{\theta})$ is reduced to the all-ones matrix (when $p = 1$ for omni-directional elements). In term of different microphone elements, CDDMA can be used as a more general framework to design CDMA. In addition, we can find that the individual control on p in (5) is possible for each microphone element, which would possibly introduce more degrees of freedom in the optimization process of CDDMA design for some specific cases. Furthermore, the proposed beamformer is not limited to first-order directional microphone. Given higher-order directional is implemented in the CDDMA , only the $\mathbf{U}(p, \boldsymbol{\theta})$ in (23) should be changed by a higher-order directional microphone response matrix.

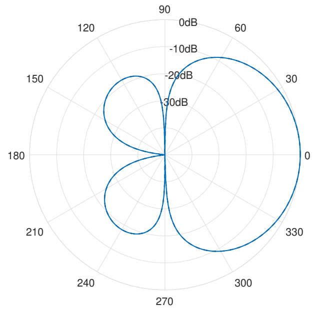
5. Design Examples

In this section, we will give some design examples of the proposed CDDMA beamformer and study their performance using the measures described in Section 3. They will be compared to CDMA obtained in [11]. In the remaining part of the paper, we assume the desired "look" direction is 0 degree, i.e., $c_0 = 1$ in (14). We take the cardioid element as the example of a directional microphone to form the CDDMA beamfomer throughout this section, i.e., $p = 0.5$ is used in (2).

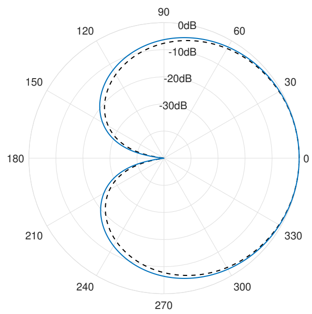
Fig.2 shows a comparison of beampatterns for two different designs between CDDMA and CDMA at frequencies of 1 kHz, 3 kHz and 6 kHz, where $r = 1.5$ cm and $M = 8$ are used. The first design is to construct a 1st-order cardioid ($c_\pi = 0$) while the second design is to build a second-order cardioid ($c_\pi = c_{\frac{\pi}{2}} = c_{\frac{3\pi}{2}} = 0$). It can be seen from the left column of Fig.2 that the beampatterns of CDDMA and CDMA are very close to the desired 1st-order cardioid at 1 kHz and 3 kHz, while at 6 kHz the CDMA beampattern deviates significantly (see Fig.2e) whereas the CDDMA beampattern still holds for the desired design. Therefore, CDDMA is more frequency-invariant than CDMA for the design of 1st-order cardioid. However, both CDMA and CDDMA for the 2nd-order cardioid are quite frequency-invariant, as depicted in the right column of Fig.2



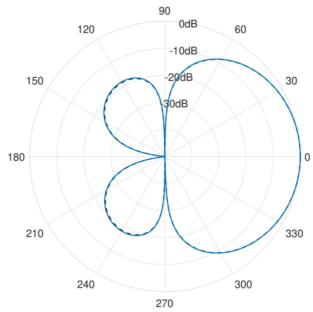
(a) 1 kHz, $c_\pi = 0$



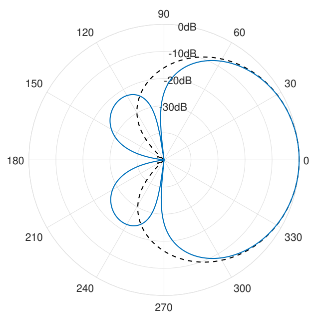
(b) 1 kHz, $c_{\frac{\pi}{2},\pi,\frac{3\pi}{2}} = 0$



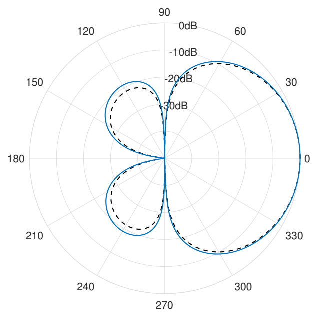
(c) 3 kHz, $c_\pi = 0$



(d) 3 kHz, $c_{\frac{\pi}{2},\pi,\frac{3\pi}{2}} = 0$

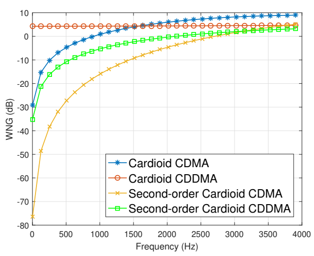


(e) 6 kHz, $c_\pi = 0$

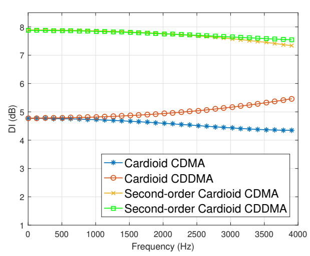


(f) 6 kHz, $c_{\frac{\pi}{2},\pi,\frac{3\pi}{2}} = 0$

Figure 2: *Beampattern comparison between robust CDMA (solid line) and robust CDDMA (dash line) for two designs: 1st-order cardioid (left column) and 2nd-order cardioid (right column)*



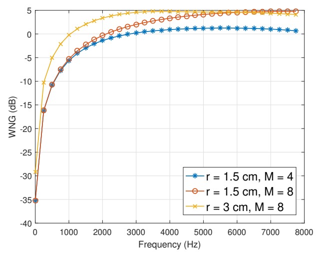
(a)



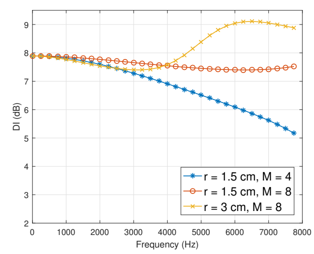
(b)

Figure 3: *WNG and DI comparison between robust CDMA and robust CDDMA for different designs*

Fig.3 shows the comparison between CDDMA and CDMA in terms of WNG and DI for the very same designs described above. We can see from Fig.3a that CDDMA for both designs exhibits a lot higher WNG as compared to CDMA at the low frequencies where WNG is usually concerned most (it is also observed that WNG of CDDMA is not as good as CDMA at the high frequencies, however, WNG at the high frequencies is usually not a concern). Meanwhile, Fig.3b indicates that CDDMA has some slight improvement in DI at the high frequencies as compared to CDMA. Lastly, for both CDMA and CDDMA, we find that the higher-order beamformers lead to the higher DI but lower WNG at the low frequencies.



(a)



(b)

Figure 4: *WNG and DI of CDDMA for different configurations*

Now let us investigate how CDDMA performs as the number of microphone elements and the radius of the array vary. Fig.4 displays the results for the second-order cardioid, where the number of microphones $M = 4$ and $M = 8$, the array radius $r = 1.5$ cm and $r = 3$ cm are attempted. We can find from the figure that the WNG at the low frequencies and DI at the high frequencies are improved when both r and M increase. For a given radius, both WNG and DI increase at the high frequencies as M increases. For a given number of microphones, it is interesting to see from Fig.4b that as r increases from $r = 1.5$ cm to $r = 3$ cm, DI decreases slightly at the low frequencies but increases noticeably at the high frequencies while the opposite is true for WNG. It should be noted that that just like CDMA there is an upper bound for the radius in order to avoid the spatial aliasing in CDDMA design (thus the case of $r = 3$ cm and $M = 4$ is omitted here).

6. Conclusions

We extend the existing work by utilizing the directional elements instead of the widely-used omni-directional elements in the CDMA design. Use of the directional microphones provides an additional degree of freedom and brings about noticeable improvements.

Given the same design constraints, we firstly show that CDDMA is more frequency-invariant than CDMA in some design. We then find out that WNG of CDDMA beamformer at the low frequencies is significantly improved, as compared to CDMA. Meanwhile, CDDMA exhibits a slight improvement in DI at the high frequencies than CDMA. In addition, similar to CDMA, we observe that the higher-order CDDMA beamformer leads to the higher DI but lower WNG at the low frequencies. We also investigate the performance of CDDMA when the array radius and number of array elements vary. WNG at the low frequencies and DI at the high frequencies are improved when both r and M increase.

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